Ec140 - Mean and Expectation

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Housekeeping

- Updated Syllabus
- Unofficial Course Capture!
- What is the weirdest concept you remember from yesterday?
- Switch to finish yesterday's slides

This Lecture

- Introduction to Data
- Mean and Expectation
- Variance and Standard Deviation

What Defines a Data Set?

- Data Set is the collection of any type information (of multiple *Datum*)
- In quantitative analysis we focus on *structured* data sets (unlike, for example, unstructured field notes).
- In econometrics the most commnon way to structure data is in tabular, or rectangular, form.
- A tabular data set is a collection of variables that with information for one or more entities.
- Entities can represent multiple individuals, one individual over time, firms, countries, etc.
- Variables are represented in columns, and observations are represented by rows. (for more on variables The Effect, Ch3)

Data

Sample of US workers (Current Population Survey, 1976)

	Wage	Education	Tenure	Female?	Non-white?
1	3.1	11	0	1	0
2	3.24	12	2	1	0
3	3	11	0	0	0
4	6	8	28	0	0
5	5.3	12	2	0	0
6	8.75	16	8	0	0
7	11.25	18	7	0	0
8	5	12	3	1	0
9	3.6	12	4	1	0
10	18.18	17	21	0	0

But What Can We Do With Data?

- We summarized it! (see the great short story by J.L. Borges on why summarizing is essential)

- One of the first thing we do when summarizing data is to look at *some type of average*.
  - Wait? *Type* of average? Isn't there just one average? called *the mean*?

But What Can We Do With Data?

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- One of the first thing we do when summarizing data is to look at *some type of average*.
  - Wait? *Type* of average? Isn't there just one average? called *the mean*?
- These is also referred as measure of central tendency.
- In this course, we will focus primarily on the mean. **From now on in this course**

average noun

Save Word

av·er·age | \ ˈa-v(ə-)rij \

Definition of *average* (Entry 1 of 3)

1 a : a single value (such as a mean, mode, or median) that summarizes or represents the general significance of a set of unequal values

b : MEAN sense 1b

2 a : an estimation of or approximation to an arithmetic mean

b : a level (as of intelligence) typical of a group, class, or series

// above the *average*

Mean

- The mean is defined by the sum of a set of values divided by the number of values.

Let's look at the mean from the "hang out with a friend" exercise.

- Total over N

$$Average(X) = \frac{1 \times 10 + 2 \times 9 + 3 \times 11}{30} = 2.03$$

- One number, **highly informative** for a variable of interest.
- Always important to keep an eye on the units and magnitude (relevant for PS1).

Mean of a Binary Variable

- The interpretation for the mean of a binary variable is different from the case when there are more than two values.

- Above, the interpretation of $Average(X) = \mathbf{2.03}$ can be read as "close to having an OK time with a friend".

- But when variables only take two values, and we assing those values to be 0 and 1, the interpretation of the mean is "the proportion of all the cases where the variable takes the value of one".

- Think of the the variable `hispanic` for students in this classroom (1 if identifies as hispanic, 0 otherwise).

Mean: Notation (Message to me: draw histogram on the board)

$$Average(X) = \frac{1 \times 10 + 2 \times 9 + 3 \times 11}{30} = 2.03$$

$$Ave(X) = 1 \times \frac{10}{30} + 2 \times \frac{9}{30} + 3 \times \frac{11}{30} = 2.03$$

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$$\overline{X}_n = x_1 \times proportion(x_1) + x_2 \times proportion(x_2) + x_3 \times proportion(x_3)$$

$$\overline{X}_n = \text{summing across all } x \; (x \times proportion_n(x))$$

$$\overline{X}_n = \sum_x x \times prop_n(x)$$

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Expected Value

- Let's look at the histogram for the exercise above (drawn in the board) and pretend it is not a sample but the entire population. How can we move from frequencies into probabilities?

- Replace frequencies by probabilities

- The population version of the sample mean is the **expected value**.

Expected Value: Definition (Discrete)

The expected value of a discrete random variable X is the weighted average of its k values $\{x_1, \ldots, x_k\}$ and their associated probabilities:

$$\begin{aligned}\mathbb{E}(X) &= x_1\,\mathbb{P}(X = x_1) + x_2\,\mathbb{P}(X = x_2) + \cdots + x_k\,\mathbb{P}(X = x_N)\\ &= \sum_x x\,\mathbb{P}(X = x)\end{aligned}$$

- Also known as the population mean.

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- Also known as the population mean. Compare it to the sample mean:

$$\overline{X}_n = \sum_x x \times prop_n(x_1)$$

Example

Rolling a six-sided die once can take values $\{1, 2, 3, 4, 5, 6\}$, each with equal probability. What is the expected value of a roll?

$$\mathbb{E}(\text{Roll}) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = 3.5.$$

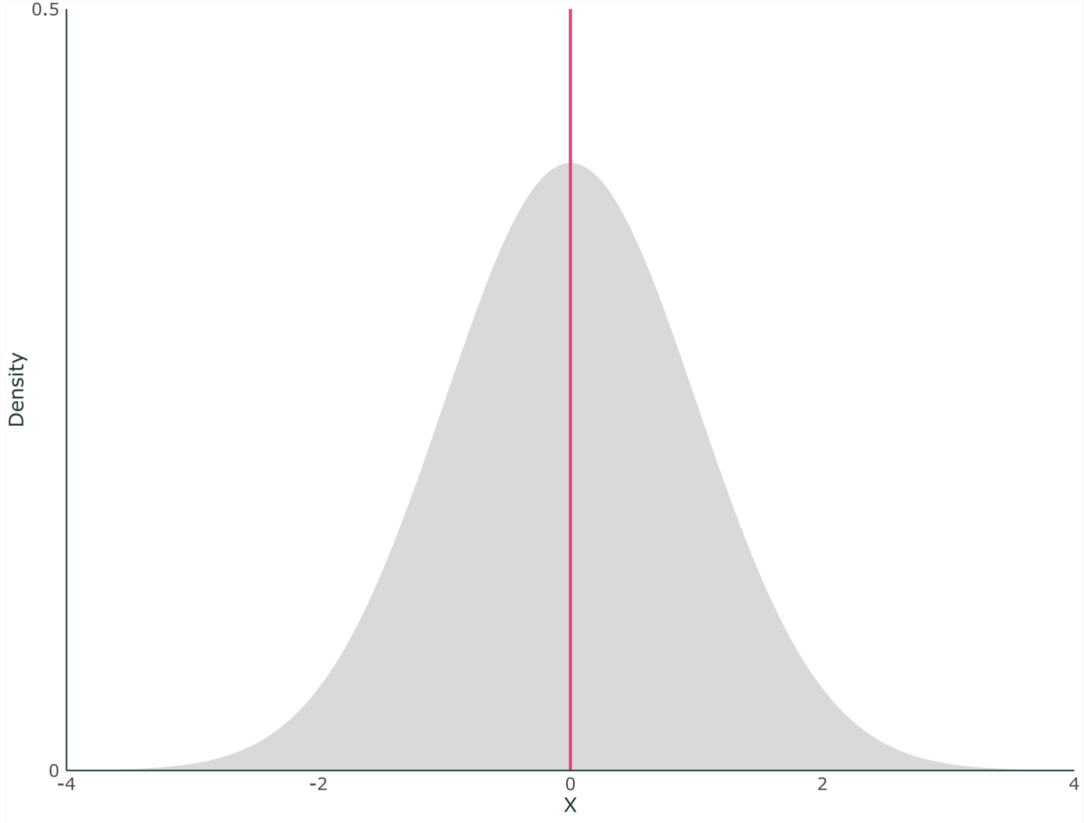
- **Note:** The expected value can be a number that isn't a possible outcome of X.

Expected Value. Definition (Continuous)

If X is a continuous random variable and $f(x)$ is its probability density function, then the expected value of X is

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x) dx.$$

- **Note:** x represents the particular values of X.



Expected Value. Definition (Continuous)

- Compare it to the discrete version
- Continuous

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x) dx.$$

- Discrete

$$\mathbb{E}(X) = \sum_{x} x f(x)$$

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- Discrete

$$\mathbb{E}(X) = \sum_{x} x f(x)$$

This explanation was inspired by
this lecture from Eddie Woo

Expected Value. Definition. One Last Thing 1/2

Let's go back to the mean of our exercise:

$$\overline{X}_n = 1 \times \frac{10}{30} + 2 \times \frac{9}{30} + 3 \times \frac{11}{30} = 2.03$$

But now let's switch the values of the random variables to: 10, 20, 30. How should we compute the mean?

$$\overline{g(X)}_n = 10 \times \frac{10}{30} + 20 \times \frac{9}{30} + 30 \times \frac{11}{30} = 20.33$$

Expected Value. Definition. One Last Thing 2/2

Hence, we can conclude, that for a random variable X, any transformation $g(X)$ has a sample aveage:

$$\overline{X}_n = \sum_x g(x) \times prop_n(x_1)$$

And an expectation:

$$\mathbb{E}(g(X)) = \sum_x g(x) f(x)$$

The same idea applies in the case of a continues random variable

Expected Value: Rules (or Properties)

Rule 1

For any constant c, $\mathbb{E}(c) = c$.

Not-so-exciting examples

$\mathbb{E}(5) = 5$.

$\mathbb{E}(1) = 1$.

$\mathbb{E}(4700) = 4700$.

Expected Value

Rule 2

For any constants a and b, $\mathbb{E}(aX + b) = a\,\mathbb{E}(X) + b$.

Example

Suppose X is the high temperature in degrees Celsius in Eugene during August. The long-run average is $\mathbb{E}(X) = 28$. If Y is the temperature in degrees Fahrenheit, then $Y = 32 + \frac{9}{5}X$. What is $\mathbb{E}(Y)$?

- $\mathbb{E}(Y) = 32 + \frac{9}{5}\,\mathbb{E}(X) = 32 + \frac{9}{5} \times 28 = 82.4$.

Expected Value

Rule 3: Linearity

If $\{a_1, a_2, \ldots, a_n\}$ are constants and $\{X_1, X_2, \ldots, X_n\}$ are random variables, then

$$\mathbb{E}(a_1 X_1 + a_2 X_2 + \cdots + a_n X_n) = a_1 \, \mathbb{E}(X_1) + a_2 \, \mathbb{E}(X_2) + \cdots + a_n \, \mathbb{E}(X_n).$$

In English, the expected value of the sum = the sum of expected values.

Expected Value

Rule 3

The expected value of the sum = the sum of expected values.

Example

Suppose that a coffee shop sells X_1 small, X_2 medium, and X_3 large caffeinated beverages in a day. The quantities sold are random with expected values $\mathbb{E}(X_1) = 43$, $\mathbb{E}(X_2) = 56$, and $\mathbb{E}(X_3) = 21$. The prices of small, medium, and large beverages are 1.75, 2.50, and 3.25 dollars. What is expected revenue?

$$\begin{aligned}\mathbb{E}(1.75X_1 + 2.50X_2 + 3.35X_n) &= 1.75\,\mathbb{E}(X_1) + 2.50\,\mathbb{E}(X_2) + 3.25\,\mathbb{E}(X_3) \\ &= 1.75(43) + 2.50(56) + 3.25(21) \\ &= 283.5\end{aligned}$$

Expected Value

Caution

Previously, we found that the expected value of rolling a six-sided die is $\mathbb{E}(\mathbf{Roll}) = \mathbf{3.5}$.

- If we square this number, we get $[\mathbb{E}(\mathbf{Roll})]^2 = \mathbf{12.25}$.

Is $[\mathbb{E}(\text{Roll})]^2$ the same as $\mathbb{E}\left(\text{Roll}^2\right)$?

No!

Expected Value

Caution

Except in special cases, the transformation of an expected value **is not** the expected value of a transformed random variable.

For some function $g(\cdot)$, it is typically the case that

$$g\left(\mathbb{E}(X)\right) \neq \mathbb{E}(g(X)).$$

Activity 1

- Let's watch another Stat 110's video. Then get together in groups of 3 and discuss:
    - Don't worry about the law of large numbers yet
    - How does the random variables becomes continuous?
    - How does linearity help with computations?